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Irreversible Thermodynamics for Stress, Strain and Other Soil Physical Property Description

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RHEOLOGIC models used to describe the mechanical behavior of soil have recently been reviewed and summarized in detail by Scott and Ko (1969). Their review (274 references) is a "state of the art" report which describes theoretical and experimental application to soils of rate process theory, granular models, and the continuum models of elasticity, plasticity, and viscoelasticity. Applications of the various models were primarily for engineering rather than agricultural purposes. Continuum models have received far more intensive recent application than granular models. Also, the application of rate process theory to soils has intensified with attempts to link particle surface properties to macroscopic soil stress--strain behavior. One subject area with nearly a complete lack of information was the effect of temperature on soil stress-strain properties.

The materials on which models have been tested were almost exclusively either clean sand or clay. Dry or totally saturated samples were generally used. The relatively small amount of work that was reported on unsaturated soil materials containing mixed particle sizes indicated difficulties were encountered obtaining reliable measurements from such samples.

Since all agricultural soils fall within the category of mixed particle size and are unsaturated when their stress-strain behavior is of most concern, they will apparently be most difficult to describe accurately with a stress-strain theory. A very flexible and possibly complex theory will be required to explain the behavior of agricultural soils.

One of the most general theoretical tools available presently is irreversible thermodynamics. M. A. Biot* in a series of 25 or more technical papers and a textbook, Biot (1965), has applied this theory to geologic and soils problems. The resulting stress-strain equations are equivalent to those obtained with general viscoelastic theory with the exception that irreversible thermodynamics allows for interactions between normally independent variables. Such interactions do not appear in classical viscoelastic theory.

Since there are major interactions between quantities in soil (stress-strain relationships and water content; water flow and heat flow; water conductivity and porosity or bulk density; salt content, water conductivity, and most likely stress-strain relationships) a theory which allows for such interactions would seem most appropriate. Taylor (1963) has applied irreversible thermodynamics to soil and biological systems with a different approach than Biot (1965). Instead of working from the most general possible case as Biot does, Taylor (1963) develops the theory from specific examples and works toward more complex cases. Such an approach aids the understanding of the origin and significance of the interaction terms for most readers. Taylor's text provides good supplemental information to Biot (1965) for anyone attempting to use irreversible thermodynamics in agriculture. The following reproduction of some of the thermodynamic relationships developed by Biot is presented to illustrate the possible application of the theory to soil stress-strain behavior and general description of soil physical properties and to point out some of the advantages and disadvantages of the theory.

THEORY REVIEW

The starting point for the thermodynamic approach is a very general isolated thermodynamic system, Biot (1954, 1955b, 1956a, 1958). V is defined as the generalized free energy of that system. A force Q_i is defined by equation [1]. The symbols i and j are indexing variables which take on integer values of 1 or greater.

$$\frac{\partial V}{\partial q_i} = Q_i \quad \ldots\ldots\ldots\ldots \quad [1]$$

A conjugate coordinate of Q_1, q_i is defined such that the product $Q_i q_i$ has the units of energy. Some examples of what Q_i and q_i can be are electromotive force and amount of electric charge flowing, chemical potential and mass of atomic species present, pressure and volume, and stress and strain.

The thermodynamic system under consideration is linear. This requires both q_i and the time derivative of q_i to be linear functions of Q_i. Equations [2] and [3] express this linearity with the constants a_i and b_i.

$$Q_i = \sum_i a_i q_i \quad \ldots\ldots\ldots\ldots \quad [2]$$

$$\frac{dq_i}{dt} = \dot{q}_i = b_i q_i \quad \ldots\ldots\ldots \quad [3]$$

Notice in equation [3] the raised dot (˙) notation for the time derivative.

If one utilizes the thermodynamic equation including temperature (T) and entropy (S_e), equation [4] for the

$$TS_e = -V + \sum_i Q_i q_i \quad \ldots\ldots \quad [4]$$

description of a general thermodynamic system and combines with it Onsagers' reciprocal relations (DeGroot and Mazur 1962), an expression called the dissipation function (D) given in equation [5] can be defined. By introducing equation [5] into equation [4], equation [6] may be obtained.

$$D = \frac{1}{2} \sum_i \sum_j b_{ij} q_i q_j \quad \ldots\ldots \quad [5]$$

Equation [6] is the general thermodynamic equation describing a system with forces acting on it and energy being lost in it. The dissipation function includes the terms that lead to the loss of energy within the system.

$$\frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i \quad \ldots\ldots\ldots \quad [6]$$

A slightly more common form of this equation is shown by equation [7].

$$\sum_j a_{ij} q_j + \sum_j b_{ij} \dot{q}_j = Q_i \quad \ldots\ldots \quad [7]$$

Equation [8] may be an even more familiar form. It may be the equation

$$C \frac{dY}{dt} + KY = F \quad \ldots\ldots\ldots \quad [8]$$

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describing a spring and dashpot in parallel or a resistor and capacitor connected in series. Constants K and C are for a spring and dashpot, respectively. Y is movement if one end of the model with the other end stationary and F is applied force. Although the basic form of equation 6 is a simple one, it may be used to describe complex phenomena because of the large number of terms in it.

To obtain solutions to this equation the operator notation of

$$A_{ij} = a_{ij} + pb_{ij} \quad \ldots\ldots\ldots\ldots [9]$$

equation [9] is used to solve equation [10]. The operator symbol for the time derivative is p. Equation [10] is identical to equation [6] with $Q_i = 0$.

$$\sum_j A_{ij} q_j = 0 \quad \ldots\ldots\ldots\ldots [10]$$

If p is considered as a parameter of the homogenous equation [10], an eigenvalue problem results. For each root — λ_s there is a solution of the form of equation [11] which is called a relaxation mode. The superscript (s) in equations [11], [12], and [13] and following equations is not to be interpreted as an exponent. The superscript (s) is an indexing variable, as are i and j. The terms $\phi_j^{(s)}$ represent constants.

$$q_j^{(s)} = \sum_s \phi_j^{(s)} e^{-\lambda_s t} \quad \ldots\ldots [11]$$

For infinite roots, the solutions take on the form of equation [12].

$$q_j^{(\infty)} = \phi_j^{(\infty)} \quad \ldots\ldots\ldots\ldots [12]$$

For zero roots the solution takes on the form of equation [13].

$$q_j^{(0)} = \phi_j^{(0)} t \quad \ldots\ldots\ldots\ldots [13]$$

Equation [12] represents pure elastic behavior, while equation [13] represents pure viscous behavior and equation [11] represents a combination of the two.

It should be pointed out that no specification has been made as to the nature of parameters that are being described by these thermodynamic equations. Only that the product $Q_i q_i$ must have units of energy.

In order to simplify solving equations where Q_i is not equal to 0, equation [6] may be normalized.

$$q_i = \sum_{j=1}^{k} Q_j \sum_s \frac{\phi_i^{(s)} \phi_j^{(s)}}{p + \lambda_s} + \phi_i^{(\infty)} \phi_j^{(\infty)} \quad \ldots\ldots\ldots\ldots [14]$$

The solution of the normalized equation is algebraic, and the transformation of the normalized coordinates back to q_i and Q_i results in equation [14].

Equation [6] may also be solved for Q_i in terms of q_j. For media that must be represented by a large number of roots, summation signs may be replaced with integrals and equation [15] results. The complicated operator in brackets can be represented by a spring, a dashpot, and an infinite series of spring-dashpot units which may be characterized by the density distribution function $\gamma(\lambda)$ and the associated constants $D_{ij}(\lambda)$. The constants $D_{ij}(\lambda)$ in equation [15] correspond to the product $\phi_i^{(s)}$. $\phi_j^{(s)}$ where s no longer represents a finite number of terms. The operator of equation [15] is of a very general form. In order to grasp the significance of these operators some specific examples are helpful.

$$Q_i = \sum^{j} \left[p \int_0^\infty \frac{D_{ij}(\lambda)\, \gamma(\lambda)}{p + \lambda} d\lambda + D_\infty + pD_o \right] q_j \quad \ldots [15]$$

For an operational stress-strain equation like equation 16, which expresses strain (ϵ) in terms of stress (σ), the operator 0^* can have many forms. One simple form is equation [17]. The term η represents the viscosity of the model, the product ηr is the elastic modulus, and p is a time derivative operator.

$$\epsilon = 0^* \sigma \quad \ldots\ldots\ldots\ldots [16]$$

$$0^* = \frac{\eta\, rp}{(r + p)} \quad \ldots\ldots\ldots\ldots [17]$$

If a constant stress of unit value is applied at time zero, [represented by the function 1(t)] the rules of operational calculus may be used to obtain the solution of the stress-strain equation in the form of equation [18]. This is the equation for the Maxwell unit (spring and dashpot in series).

$$\epsilon = (1/\eta r + t/\eta)\, 1(t) \quad \ldots\ldots [18]$$

Another common operator is of the form of equation [19].

$$0' = \beta p^\theta \quad \ldots\ldots\ldots\ldots [19]$$

The terms β and θ are constants. For the same constant stress loading conditions as in the previous example, the solution for this operator is given in equation [20]. The gamma function is represented by $\Gamma(1 + \theta)$. The strain represented by this equation is similar to that of a Voight model (spring and dashpot in parallel). θ varies between 0 and 1. If $\theta = 0$ the response is elastic. If $\theta = 1$ the response is viscous. For $0 < \theta < 1$ response is viscoelastic.

$$\epsilon = [t^\theta / \beta\Gamma(1 + \theta)]\, 1(t) \quad \ldots\ldots [20]$$

A wide variety of material behavior may be described by using either of these two operators or simple additive combinations of them.

Operator $0'$, equation [19], can be rewritten to take the general form of the operator in equation [15] as in equation [21].

$$0' = \frac{\beta \sin\theta\, \pi}{\pi} \int_0^\infty \frac{p}{p + \lambda} \lambda^{(\theta - 1)} d\lambda \quad \ldots\ldots\ldots\ldots [21]$$

The corresponding spectral distribution function is given in equation [22].

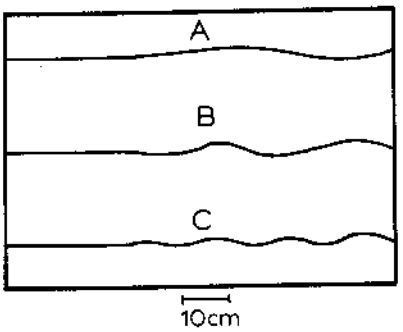
$$D_{ij}(\lambda)\, \gamma(\lambda) = \frac{\beta \sin\theta\, \pi}{\pi} \lambda^{(\theta - 1)} . [22]$$

$D_{ij}(\lambda)$ in this case is not a function of λ. It is written as it is to show the correspondence with equation [15]. D_∞ and D_o are zero.

Note that equation [16] has the same form as the simple stress-strain equation for one-dimensional elastic theory. Biot has established what is called a correspondence principle. The correspondence principle states that viscoelastic operators may be substituted directly for the elastic moduli of elastic theory equations. This is the only requirement necessary to convert the equations of elastic theory to those of viscoelastic theory (within the limits of applicability of viscoelastic theory). A person may therefore attack a mechanics problem and obtain descriptive equations with elastic theory, substitute operators for the elastic constants, and solve to obtain a viscoelastic solution for the problem.

APPLICATIONS

For materials that may be described satisfactorily with simple operators or distribution functions, as equation [22], Biot (1962a and 1962b) suggests the analysis of "internal" properties of a material by dynamic testing. If the behavior of interest, whether it be heat flow, water flow, salt movement or strain, exhibits viscoelastic (time dependent) behavior, the measurement of that property will be dependent upon the frequency of the imposed driving force or forces during the measurement. Consider the interaction of compaction and water flow in an unsaturated soil. If stress at one end of an unsaturated soil column is cycled more rapidly than



Folding Instability of Layered Media

FIG. 1 Tracing of a photograph showing the buckling of a 1-mm thick acetate strip in corn syrup with compressive loads on the acetate strip of 1.6 kg (A), 6.6 kg (B), and 11.6 kg (C) (Biot, Ode and Roever, 1961).

the stress can be redistributed from both the soil and water in the column to just the soil, the column will probably have different properties than if a constant stress were applied. At frequencies higher than those permitting stress redistribution by water flow, other relaxation mechanisms that respond more rapidly than water flow can be examined. Such a measurement should be essentially independent from water flow, since the water would be essentially "frozen" in place by cyclic stresses too rapid to be redistributed by flow. To expect distinct non-interacting frequencies for each stress redistribution mechanism in a soil would not be reasonable, of course. Such an approach, however, may help to explain some of the confusing responses of soils to vibrating loads.

Biot (1941, 1955a, 1956b, 1956c, 1963) and Biot and Willis (1957) develop equations which describe the movement of both the solid and fluid material of a saturated porous medium. He presents very general equations using a three-dimensional elastic theory analysis and a generalized form of Darcy's law to include all possible cases of nonuniformity. Simplifications are illustrated as various types of symmetry are considered. He also considers the case of variable permeability where permeability is dependent upon the amount of deformation of the solid phase of the porous medium. His solutions, however, are very general. There are no specific numerical examples given. He also illustrates the use of the equations by analyzing the settlement of a loaded soil column. The solution of this problem is also carried out in very general terms.

Cyclic driving "forces" during testing need not be limited to mechanical stress to apply the viscoelastic analysis. One of the many problems that will, no doubt, be encountered in such tests will be interactions of different phenomena. Heat flow, water movement in both liquid and vapor phase, and soil movement may have to be considered simultaneously. Salt effects on soil stress-strain or other properties might be studied using electro-osmosis for control of salt and water distribution in a column. The construction of apparatus to handle simultaneous measurement or imposition of stress, pore water pressure, temperature, salt concentration or voltage, all at selectable frequencies, may be a formidable task.

Irreversible thermodynamics provides a theoretical framework within which such a large number of variables may be examined and interactions between them permitted. When a large number of variables are considered, the number and length of equations required may approach unmanageable quantities. For example, if only stress and strain, heat flow and water flow in both liquid and vapor phase are considered for a general three-dimensional analysis, at least 12 simultaneous equations with 12 unknowns would have to be solved. As many as 144 individual constants have to be evaluated for the most general case. Judicious selection of experimental conditions will be necessary to minimize the number of measurements required to evaluate interactions. Selections of boundary conditions and symmetry of samples and test equipment are used to reduce the number of measurements in any one experiment.

External or Gross Behavior

Another interesting area of application of viscoelastic theory has been developed by Biot et al. (1961), Biot and Ode (1962), and Biot (1959a, 1959b, 1961, 1963, 1964). None of this theoretical development will be reproduced here, however. The topics developed are the folding of stratified viscoelastic media and internal buckling of confined multilayered structures. Although primary application of this theory was to geologic rock structures, the discussions of the nature of internal buckling and ultimate failure of viscoelastic layers in response to lateral confining pressure may provide a starting point for the description of soil behavior during tillage. He is able to confirm some of his predicted folding behavior for thin plastic and aluminum sheets imbedded in viscous materials such as corn syrup. In Fig. 1 is illustrated the general type of buckling that occurs in thin strips of viscoelastic

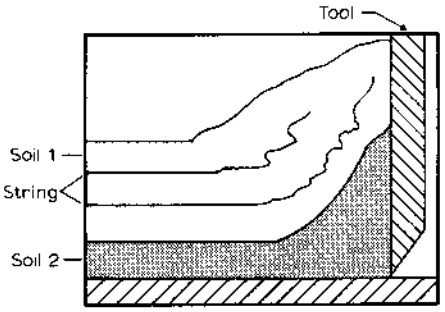


FIG. 2 Tracing of a photograph showing the deformation of soil in front of a simple tillage tool (Gill and Vandenberg, 1967).

media. When ratios of viscosities or the time constants of adjacent layers are nearly equal, the folds that occur are not as prominent. Such would most likely be the case in soils. However, a buckling type of failure appears to be occurring in front of a simple tillage tool as shown by Gill and Vandenberg (1967) and shown in Fig. 2. Conditions can exist where buckling phenomena is not exhibited, however, failure or shear planes still occur. On this subject, Biot (1965) discusses the relation of internal instability to the occurrence of sliplines or the shear planes in elastic or "plastic" media.

In order to study folding behavior more completely, additional theoretical development must be used. This involves including inertia terms in the general descriptive equations. Equation [23] gives the form of the necessary equation. W is a quadratic function similar in form to D. Biot (1962a, 1962b) applied this equation to such topics as sonic and ultrasonic wave transmission in porous media

$$\frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} + \frac{d}{dt}\left[\frac{\partial W}{\partial \dot{q}_i}\right] = Q_i \qquad [23]$$

and dynamic stability of multilayered media. Equation [23] is a generalized form of the common mass-spring-dashpot or resistor-capacitor-inductor equation. The application of this theory to thermoelastic phenomena provides behavior equations which relate deformation and heat flow and are shown to be directly analogous to the description of consolidation and fluid flow in an elastic porous medium, Biot (1956a).

Eringen (1960), Valanis (1968), and Gorodtsov and Leonov (1968) have also derived the fundamental behavioral equations for fluids, solids, and viscoelastic material by utilizing irreversible thermodynamics. Their work parallels part of that of Biot's. Schapery (1964)

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continued from the development of behavioral equations by Eringen (1960) and presented an analysis of crack propagation in viscoelastic media. His work includes as a simplified case the equivalent to Griffith's (1920) criterion for initiation of crack growth in an elastic material. As with the work of Biot, this material is all theoretical and untested, particularly for soil. The vast scope of the theory, however, makes its application appear promising.

Non-equilibrium thermodynamics were used specifically for the evaluation of tillage tool-soil interaction by Fornstrom, Brazee, and Johnson (1970). They have taken a major first step in the use of this theory for soil by describing a specific system, measuring its response, and examining the nature of the force function required to provide the observed response.

PROJECTIONS AND EVALUATION

If one is to compare classical viscoelastic theory with that based on thermodynamic development, one can find parallel descriptions for almost any form of material behavior to which the models are applicable. As far as practical application is concerned, the general viscoelastic theory has been utilized to a much greater extent and is much more familiar to more people. Procedures are complete for most of the measurements required by viscoelastic theory. This is not the case for the thermodynamic theory. Experiments designed to evaluate it will be inherently more complex because of the interrelationships between variables that must be evaluated. Evaluation of mechanisms of behavior should be equally well described by either approach. Additional studies, such as those of Wu, et al. (1966), who used rate process theory to analyze soil consolidation, should contribute to the understanding of mechanisms of soil behavior.

With these theories a description of the influence of water content on soil strength and behavior should be possible. Krizek (1968) has suggested that the effects of temperature on polymer behavior may be very similar to the influence of water content on soil behavior. He has suggested that the "glass temperature" of plastics (that temperature range over which plastics convert from primarily solid to primarily liquid properties) may correspond to the shrinkage limit of clays, for example. The implication is that some soils will exhibit no or very little shear strength when wet above a certain critical water content. Field observations of earthslides where surface soil separates from that beneath it along a saturated horizon and formation of crusts in the bottom of irrigation furrows are consistent with such predictions. Also implied is the absence of the slides or crusts if water content can be maintained below the critical level.

For plastics there is a time-temperature correspondence such that a high temperature will result in deformation expected at a lower temperature with longer loading time. If temperature effects on plastic are indeed similar to water content effects in soils, this may imply a time-water content correspondence. Water content influences on soil strength might be studied by long time experiments at constant water content or long time behavior may be studied by varying (increasing) water content of a sample.

Comments on the relationship between mathematical models and physical reality may be in order at this time. Kolsky (1964) has pointed out that true agreement between mathematical models (particularly the simpler ones) and physical reality will be rare. The only way to determine actual behavior of a material is to measure it. In many uses however, a model will be found that matches real behavior sufficiently for the purposes at hand.

The process of relating physical mechanisms to observed phenomena, such as is proposed by the thermodynamic approach and rate process theory, is subject to debate. Wei (1966) for example states that in spite of thermodynamic origins of equations they are simply phenomenological equations (just fancy best-fit equations) that can be adapted to describe any combination of linear processes. Nonlinear behavior is not accounted for and no mechanisms can be implied from the theory. Others do not consider these restrictions to exist for all applications of the theory. Actual data correlation attempts will have to be made to help resolve such debates.

The irreversible thermodynamic theory should be kept in proper perspective, however. It is not a cure all, no matter what the extent of its possible application. It is a linear theory as is elastic theory. It has the capacity to describe very complex, combined phenomena but if nonlinear processes are involved it must be modified to account for them. Otherwise, it should not be expected to provide more than a reference with which to compare observed behavior just as elastic theory provides a reference with which simple nonlinear behavior may be compared. Procedures for adaption of the theory to nonlinear processes have been suggested by Eringen (1960) and Biot (1965).

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